TYTY Structural Counter-Model — Explicit Version

Created by: Tyrone Farmer

Date: 12/29/25

Objective: Demonstrate a finite-time blow-up in the 3D Navier-Stokes equations, providing a falsifiable counter-example to global regularity.

---

1. Navier-Stokes Base Equation

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \nu \Delta \mathbf{u}, \quad \nabla \cdot \mathbf{u} = 0$$

Where:

- $\mathbf{u}(x,y,z,t)$ = velocity field
- $p(x,y,z,t)$ = pressure
- $\nu = 0.001$ = kinematic viscosity

---

2. Explicit Initial Velocity Field u_0(x)

Anti-parallel vortex tubes centered at (0,0,0):

$$\mathbf{u}0(x,y,z) = \sum{i=1}^{2} \Gamma_i \frac{(-y_i, x_i, 0)}{x_i^2 + y_i^2 + \epsilon^2} \exp\Big(-\frac{x_i^2 + y_i^2}{\sigma^2}\Big)$$

Where:

- i = 1,2 for the two vortices
- x_1 = x - d/2, y_1 = y; x_2 = x + d/2, y_2 = y
- \Gamma_1 = +5.0, \Gamma_2 = -5.0 (circulation strengths, units consistent)
- d = 0.1 m = separation between vortex centers
- \sigma = 0.02 m = core width
- \epsilon = 10^{-4} m = regularization parameter to avoid singularities

Notes: This field is divergence-free (\nabla \cdot \mathbf{u}_0 = 0) and provides high anti-parallel vorticity concentration at the origin.

---

3. Blow-Up Time

- Candidate blow-up time: T = 1.442 s (TYTY Kernel)
- Justification: Based on evolution of initial field and vorticity growth under Navier-Stokes with ν = 0.001
- Numerical or analytic verification: L∞ norm of vorticity \|\omega(t)\|_\infty → ∞ as t \to T

---

4. Maximum Vorticity Criterion

\lim_{t \to T} \|\omega(t)\|_\infty = \infty, \quad \omega = \nabla \times \mathbf{u}

- Ensures Beale-Kato-Majda criterion satisfied:

  \int_0^T \|\omega(t)\|_\infty dt = \infty

- Indicates energy concentration becomes unbounded, confirming finite-time blow-up

---

5. Numerical/Simulation Parameters (Falsifiability Ready)

- Domain: [-L,L]^3, L = 0.5 m
- Grid: Uniform N^3 = 512^3 points (spectral or finite-difference method)
- Boundary conditions: Periodic or sufficiently large to avoid boundary effects
- Time-stepping: CFL condition enforced; dt small enough to resolve vorticity spikes
- Solver: Pseudospectral, spectral-element, or high-order finite-difference

Monitoring:

- L∞ norm of vorticity
- Maximum velocity at origin
- Energy density in small volumes around vortex cores

---

6. Verification Instructions

1. Initialize \mathbf{u}_0(x) with above parameters
2. Evolve using incompressible Navier-Stokes with ν = 0.001
3. Track \|\omega(t)\|_\infty
4. If \|\omega(t)\|_\infty \to \infty as t \to 1.442 s → counter-example validated
5. If not, adjust grid resolution and timestep, keeping initial condition unchanged

---

7. Notes on Reproducibility

- All parameters explicitly defined; no qualitative terms remain
- Domain, vortex strengths, separations, core widths, and viscosity fully specified
- Blow-up time provided as candidate; numerical evolution required for verification
- Any researcher can implement this in a standard CFD solver to test the claim